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LOSSES IN FOUNTAIN-EFFECT PUMPS

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Three loss mechanisms in fountain-effect pumps are identified and analyzed. Two of these mechanisms reduce the mechano-caloric effect, by reducing the mass transferred per unit heat input. The first loss mechanism of these losses is the result of normal fluid leakage through the pump. The second loss mechanism is the result of the finite separation between the porous plug and the heater. The third loss mechanism reduces the thermomechanical effect. It results from the Gorter-Mellink interaction within the porous plug. All three of these loss mechanisms are shown to reduce pump efficiency. They are then applied to an example to illustrate the relative significance of the three mechanisms.

INTRODUCTION

Fountain-effect pumps are being considered for transferring large quantities of liquid helium between Dewars in space.¹ A fountain-effect pump consists of a porous plug and a heater. Ideally the porous plug would be a "superleak" through which the superfluid component flows, the normal fluid component is immobilized, and the first critical velocity is not exceeded. Deviations from this ideal behavior result in losses that reduce the pump's efficiency. Three such mechanisms are discussed here.

The transport of helium II through an ideal superleak is accompanied by a heat flow

$$\dot{Q} = -\dot{m}TS \quad (1)$$

This is the mechano-caloric effect² in which \dot{m} is the mass flow rate, T is the temperature, and S is the absolute entropy. The derivation of (1) assumes that only the superfluid component moves through the superleak. In a practical fountain-effect pump, the normal fluid can also flow. This normal fluid flow is a loss mechanism that affects the heat transport. The heat transport is further affected by the finite separation between the porous plug and the heater. This is the second loss mechanism.

The pressure head developed by a fountain-effect pump is given by the London relation² (also called the thermomechanical effect or fountain effect)

$$\Delta P_p = \rho S \Delta T \quad (2)$$

where ρ is the density and ΔT is the temperature difference across the pump. This relation is based on the assumption that the fluid velocity within the pump is below the first critical velocity, v_c . In the Gorter-Mellink flow regime above the first critical velocity, (2) must be modified, resulting in the third loss mechanism.

The efficiency (the ratio of the hydraulic power produced by the pump to the total input power) of a fountain-effect pump is³

$$\epsilon \approx \dot{m} \Delta P_p / \rho \dot{Q} \quad (3)$$

Substituting in (1) and (2) yields

$$\epsilon \approx S \Delta T / S_o T_o \quad (4)$$

where T_o and S_o are the temperature and entropy at the outlet of the porous plug. Since the efficiency of the pump depends on both (1) and (2), the efficiency will also be affected by the loss mechanisms that change these two equations.

NORMAL FLUID LEAKAGE

Consider two volumes of liquid helium II (Fig. 1) at the same temperature, T , and pressure, P . Thus the superfluid density, ρ_s , the normal fluid density, ρ_n , and the total density, $\rho = \rho_s + \rho_n$, are the same in both volumes. If a mass flow, \dot{m}_s , of pure superfluid flows from V_1 to V_2 isothermally and isobarically, then, by (1), a heat flux, \dot{Q}_s , must flow in the reverse direction

$$\dot{Q}_s = -\dot{m}_s TS \quad (5)$$

This can be thought of as the heat of converting helium from an equilibrium ratio of ρ_s/ρ_n to pure superfluid in V_1 and, conversely, it is the heat of conversion of the reverse process in V_2 .

If instead, a mass flux, \dot{m} , with an equilibrium ratio of ρ_s/ρ_n were to flow, there would be no heat of conversion. Thus the heat flux would be

$$\dot{Q}_e = 0 \quad (6)$$

In this case the mass flow can be thought of as the superposition of a pure superfluid flow $\dot{m}_s = \dot{m} \rho_s / \rho$, and of a pure normal fluid flow, $\dot{m}_n = \dot{m} \rho_n / \rho$. These two flows result in the heat fluxes \dot{Q}_s and \dot{Q}_n , respectively. Since $\dot{Q}_e = \dot{Q}_s + \dot{Q}_n$, then, by (6), (5) and the relations between \dot{m} , \dot{m}_s , and \dot{m}_n , we find

$$\dot{Q}_n = \dot{m}_n TS \rho_s / \rho_n \quad (7)$$

This is the mechano-caloric effect for pure normal fluid flow.

In the generalized case, the net heat flux will be

$$\dot{Q} = -\dot{m}_s TS + \dot{m}_n TS \rho_s / \rho_n \quad (8)$$

where the net mass flow is $\dot{m} = \dot{m}_n + \dot{m}_s$. This can be written as:

$$\alpha_1 \dot{Q} = -\dot{m} TS \quad (9)$$

where α_1 is the loss factor

$$\alpha_1 = [1 - (\dot{m}_n / \dot{m})(\rho / \rho_n)]^{-1} \quad (10)$$

Thus, if the normal fluid flows, the net mass flow per unit heat flux is reduced by a factor α_1 .

For a fountain-effect pump, the loss factor can be expected to depend on the flow regime within the pump. Consider the transfer system shown in fig. 2. The helium is being transferred from the supply to the receiver. Both tanks are at saturated conditions; T_1, P_1 and T_2, P_2 , respectively. The pump is mounted in the wall of the supply tank and connected to the receiver by a long transfer line. The pump will be modeled as a bundle of n parallel tubes of length l and diameter d , where $d \ll l$. The pump pressure head, for flows below v_c , is given by (2). In this flow regime, the normal fluid flow is laminar. This flow can be described by the Poiseuille equation.³

$$\Delta P_p = -Z_p \dot{m} \eta / \rho_n \quad (11)$$

where $Z_p = (128 l) (\pi d^4 n)^{-1}$ is the impedance of the pump and η is the viscosity. If the flow within the transfer line is laminar, the net mass flow is determined by

$$\Delta P = Z_l \dot{m} \eta / \rho \quad (12)$$

where $\Delta P = \Delta P_o + \Delta P_p$, $\Delta P_o = P_1 - P_2$, and Z_l is the transfer line impedance. Otherwise, if the flow is turbulent⁴

$$\rho \Delta P = (Z_t \dot{m} \eta^{1/7})^{7/4} \quad (13)$$

where Z_t is the impedance. Both of these impedances depend on the heat load on the transfer line and on its geometry.⁵ Substituting (11) and (12) or (13) into (10) yields

$$\alpha_1 = [1 + (Z/Z_p) \kappa]^{-1} \quad (14)$$

where $Z = Z_l$ and $\kappa = (1 + \Delta P_o / \Delta P_p)^{-1}$ for laminar flow, while $Z = Z_t$ and $\kappa = (\rho \Delta P_o / \eta)^{2/3} (1 + \Delta P_o / \Delta P_p)^{-4/9} \Delta P_p$ for turbulent flow. Thus, α_1 is a measure of the impedance mismatch in the system. It is greatest when $Z_p \gg Z$.

HEATER POSITION

In discussing the effect of the heater position, we will neglect any heat that is conducted from the transfer line back to the pump. This assumption is reasonable because forced convection is expected to dominate the heat transport within the transfer line.⁴ The pump is modeled (fig. 3) as a porous plug separated from the heater by a gap of distance h . The plug, the heater, and the space in between all have a cross-sectional area (perpendicular to the flow) of a . The heat generated by the heater, \dot{Q} , will be carried to the porous plug by conduction within the gap, \dot{Q}_k , and carried downstream by convection, \dot{Q}_c , with $\dot{Q} = \dot{Q}_k + \dot{Q}_c$. \dot{Q}_k is also the heat that makes the pump work; therefore

$$\dot{Q}_k = -\dot{m} S T \quad (15)$$

and

$$\dot{Q}_c = \dot{m} C_p \delta T \quad (16)$$

where C_p is the heat capacity, and δT is the temperature difference between the liquid at the plug's exit and at the heater. If the flow in the gap is laminar,³

$$\dot{Q}_k = -\rho^2 S^2 T a^2 \delta T (8 \pi \eta h)^{-1} \quad (17)$$

Combining (15), (16), and (17) yields

$$\alpha_2 \dot{Q} = -\dot{m}ST \quad (18)$$

where

$$\alpha_2 = [1 + (8\pi\eta/\rho)(h/a^2)\dot{m}\beta]^{-1} \quad (19)$$

is the loss factor and $\beta = C_p/\rho S^2 T$. At high flow rates, in the turbulent flow regime, (17) becomes³

$$\dot{Q}_k = -\rho_s STa(S\delta T/A\rho_n h)^{1/3} \quad (20)$$

where A is the Gorter-Mellink constant. This results in a loss factor of the same form as (19) but with η replaced by an effective viscosity

$$\eta_{eff} = aA'\dot{m}^2/8\pi a \quad (21)$$

where $A' = A\rho\rho_n/\rho_s^3$. In either case, a gap between the heater and the plug reduces the net mass flow per unit heat input, and this loss increases with separation. A small gap and a large cross-sectional area is required for α_2 to approach 1.

REDUCTION OF THE THERMOMECHANICAL EFFECT

The discussion on normal fluid leakage assumed only that the normal fluid flow is laminar. This assumption remains valid above v_c , in the Gorter-Mellink flow regime. Thus the derivation of α_1 remains valid in this regime. However, the thermomechanical effect (2) is affected. It becomes³

$$\Delta P_p = \rho SAT - A\rho\rho_n(\dot{m}_s/\rho_s - \dot{m}_n/\rho_n)^3(\xi a)^{-3} \quad (22)$$

where ξ is the porosity ($\xi a = n\pi d^2/r$). This can be written in the form

$$\Delta P_p = \alpha_3 \rho SAT \quad (23)$$

If the flow within the transfer line is laminar, then substituting (11), (12), and (22) into (23) yields the loss factor

$$\alpha_3^{-1} = 1 + (\rho/\eta\xi a Z_l)^3(\Delta P_p)^2(\gamma + \Delta P_o/\Delta P_p)^3 \quad (24)$$

where $\gamma = (1 + Z_l/Z_p)$.

If the flow within the transfer line is turbulent, then substituting (11), (13), and (22) into (23) results in

$$\alpha_3^{-1} = 1 + A'(\rho/\eta\xi a Z_t)^3(\Delta P_p)^2[(\eta^2/\rho\Delta P_p)^{3/7}(1 - \Delta P_o/\Delta P_p)^{4/7} + Z_t/Z_p]^3 \quad (25)$$

Equations (24) and (25) apply only if the flow within the porous plug is greater than v_c . Below v_c , there is no mutual friction, so there $\alpha_3 \equiv 1$.

DISCUSSION

All of these loss factors affect the pump's efficiency. If the loss factors are included by substituting (9), (18), and (23) into (3),

$$\epsilon \approx \alpha S \Delta T / S_0 T_0 \quad (28)$$

is the reduced efficiency where $\alpha = \alpha_1 \alpha_2 \alpha_3$.

As an example, we will evaluate these losses for a hypothetical transfer system containing a porous plug similar to the one discussed in Ref. 1 and with a transfer line similar to the one discussed in Ref. 5. The parameters of this system are summarized in table 1. The worst-case flow situation will be considered. This has Gorter-Mellink flow within the plug. The flow within the transfer line and in the plug-heater gap will be fully developed turbulence. This results in $\alpha_1 = 0.996$, $\alpha_2 = 0.920$, $\alpha_3 = 0.131$; therefore, $\alpha = 0.120$ and $\epsilon = 0.0035$. Thus, in this example, these losses are clearly significant. This example illustrates the importance of choosing a porous plug to keep the peak flow within the plug below v_c . Keeping the flow below v_c raises α_3 to 1, which would raise α to 0.916, in this example. The flow regime in the plug-heater gap appears to be less important. However, care should be taken to keep the gap small and the cross-sectional area of the plug and heater large. Finally, the transfer line impedance should be kept much smaller than the impedance of the plug. Finally, the various loss mechanism discussed here were analyzed assuming that the temperature difference across the pump is small. If the temperature difference is significant, then appropriate integrations must be done, as was done by Ref. 6 for the loss caused by normal fluid leakage at high pressure heads.

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TABLE 1 Transfer system parameters

Quantity	Symbol	Value
Temperature	T	1.8 K
Net mass flow	m	0.04 kg/s
Plug-heater gap	h	0.01 m
Cross-sectional area	a	$4 \times 10^{-3} \text{ m}^2$
Porosity	ξ	0.38
Plug impedance	Z_p	$2.3 \times 10^{12} \text{ m}^{-3}$
Line impedance	Z_t	$4.5 \times 10^4 \text{ m}^{-3}$
Plug pressure head	ΔP_p	$5.2 \times 10^2 \text{ Pa}$
Line pressure drop	ΔP	$3.6 \times 10^2 \text{ Pa}$

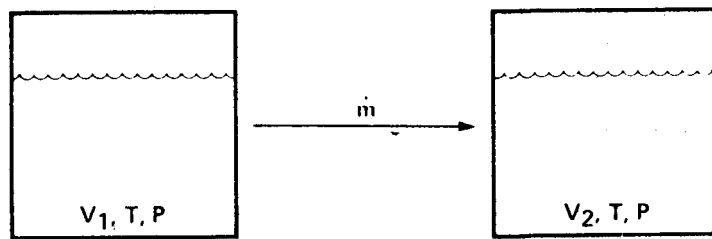


Fig. 1 The isothermal isobaric transfer of liquid helium II between two containers.

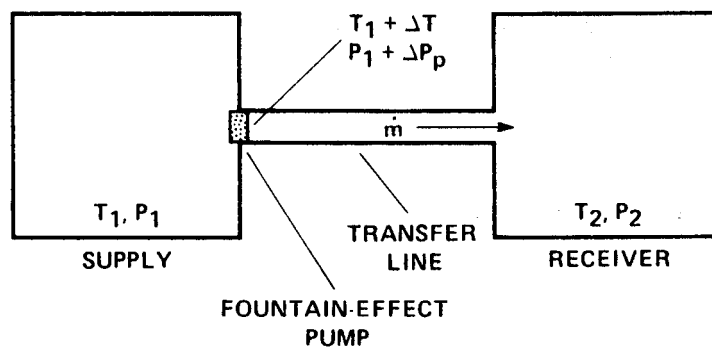


Fig. 2 The transfer of liquid helium II between two containers through a transfer line, using a fountain-effect pump.

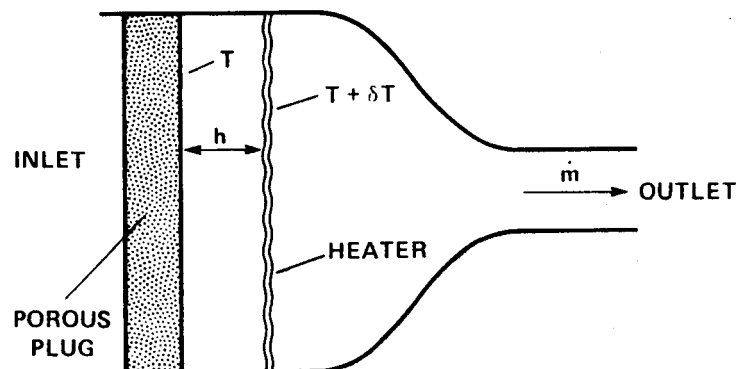


Fig. 3 A fountain-effect pump showing the heater separated from the porous plug by a distance h . The plug, heater, and volume of liquid between are all

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